(iii) $f(x) = x$, x is a real number.

Miscellaneous Examples

Example 18 Let **R** be the set of real numbers. Define the real function

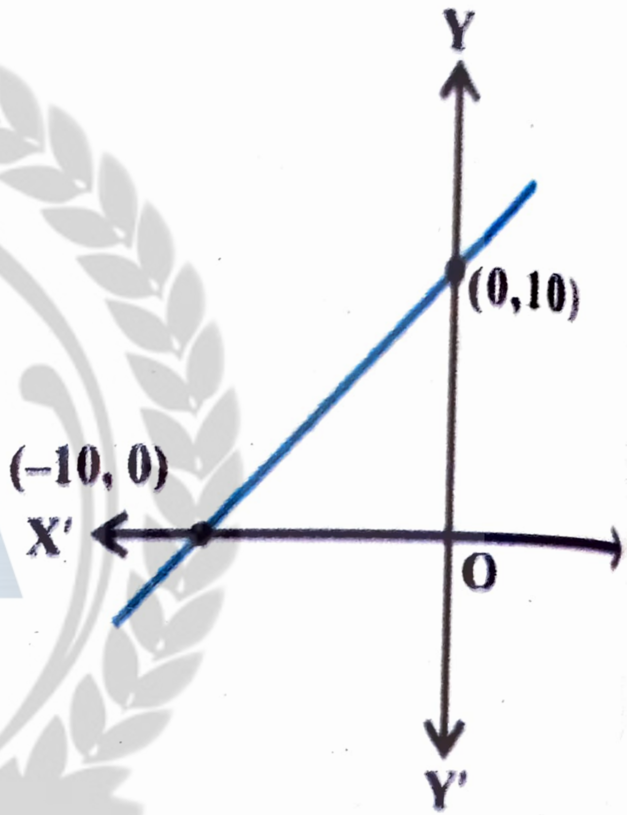
$$f: \mathbf{R} \rightarrow \mathbf{R} \text{ by } f(x) = x + 10$$

and sketch the graph of this function.

Solution Here $f(0) = 10, f(1) = 11, f(2) = 12, ..., f(10) = 20$, etc., and

$$f(-1) = 9, f(-2) = 8, ..., f(-10) = 0 \text{ and so on.}$$

Therefore, shape of the graph of the given function assumes the form as shown in Fig 2.16.



$f(x) = x + 10$

Fig 2.16

Remark The function f defined by $f(x) = mx + c$, $x \in \mathbf{R}$, is called *linear function*, where m and c are constants. Above function is an example of a *linear function*.

Example 19 Let R be a relation from **Q** to **Q** defined by R = $\{(a,b): a,b \in$ **Q** and $a - b \in$ **Z**$\}$. Show that

(i) $(a,a) \in$ R for all $a \in$ **Q**

(ii) $(a,b) \in$ R implies that $(b, a) \in$ R

(iii) $(a,b) \in$ R and $(b,c) \in$ R implies that $(a,c) \in$ R

Solution (i) Since, $a - a = 0 \in$ **Z**, if follows that $(a, a) \in$ R.

(ii) $(a,b) \in$ R implies that $a - b \in$ **Z**. So, $b - a \in$ **Z**. Therefore, $(b, a) \in$ R

(iii) (a, b) and $(b, c) \in$ R implies that $a - b \in$ **Z**. $b - c \in$ **Z**. So, $a - c = (a - b) + (b - c) \in$ **Z**. Therefore, $(a,c) \in$ R

Example 20 Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function from **Z** into **Z**. Find $f(x)$.

Solution Since f is a linear function, $f(x) = mx + c$. Also, since $(1, 1), (0, -1) \in$ R, $f(1) = m + c = 1$ and $f(0) = c = -1$. This gives $m = 2$ and $f(x) = 2x - 1$.

Example 21 Find the domain of the function $f(x) = \dfrac{x^2 + 3x + 5}{x^2 - 5x + 4}$

Solution Since $x^2 - 5x + 4 = (x - 4)(x - 1)$, the function f is defined for all real numbers except at $x = 4$ and $x = 1$. Hence the domain of f is **R** $- \{1, 4\}$.

Example 22 The function f is defined by

$$f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

Draw the graph of $f(x)$.

Solution Here, $f(x) = 1 - x, x < 0$, this gives

$f(-4) = 1 - (-4) = 5$;

$f(-3) = 1 - (-3) = 4$,

$f(-2) = 1 - (-2) = 3$

$f(-1) = 1 - (-1) = 2$; etc,

and $f(1) = 2, f(2) = 3, f(3) = 4$

$f(4) = 5$ and so on for $f(x) = x + 1, x > 0$.

Thus, the graph of f is as shown in Fig 2.17

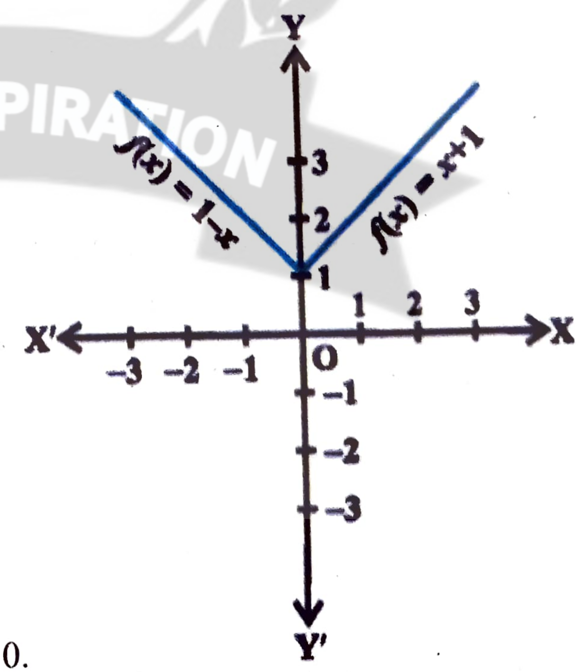


Fig 2.17

Summary

In this Chapter, we studied about relations and functions. The main features of this Chapter are as follows:

- ***Ordered pair*** A pair of elements grouped together in a particular order.
- ***Cartesian product*** A × B of two sets A and B is given by

  $A \times B = \{(a, b): a \in A, b \in B\}$

  In particular $\mathbf{R} \times \mathbf{R} = \{(x, y): x, y \in \mathbf{R}\}$

  and $\mathbf{R} \times \mathbf{R} \times \mathbf{R} = (x, y, z): x, y, z \in \mathbf{R}\}$
- If $(a, b) = (x, y)$, then $a = x$ and $b = y$.
- If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- $A \times \phi = \phi$
- In general, $A \times B \neq B \times A$.
- ***Relation*** A relation R from a set A to a set B is a subset of the cartesian product A × B obtained by describing a relationship between the first element x and the second element y of the ordered pairs in A × B.
- The ***image*** of an element x under a relation R is given by y, where $(x, y) \in R$,
- The ***domain*** of R is the set of all first elements of the ordered pairs in a relation R.
- The ***range*** of the relation R is the set of all second elements of the ordered pairs in a relation R.
- ***Function*** A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B.

  We write f: A→B, where $f(x) = y$.
- A is the domain and B is the codomain of f.

- The range of the function is the set of images.
- A real function has the set of real numbers or one of its subsets both as its domain and as its range.
- ***Algebra of functions*** For functions $f : X \to \mathbf{R}$ and $g : X \to \mathbf{R}$, we have

$$(f + g)(x) = f(x) + g(x),\ x \in X$$

$$(f - g)(x) = f(x) - g(x),\ x \in X$$

$$(f.g)(x) = f(x)\,.g(x),\ x \in X$$

$(kf)(x) = k\,f(x)\,),\ x \in X$, where k is a real number.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ x \in X,\ g(x) \neq 0$$